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Coherent propagation dynamics of an adiabatic four-waveguide directional coupler: a generic approach

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Abstract. A generic model is presented to explore optical analogues of coherent population transfer and trapping in a four-waveguide (WG) directional coupler. In contrast to conventional counterintuitive order for the coupling coefficients, the present model highlights the robustness of the approach irrespective of any particular coupling order with varying conditions of initial light distribution. The coherent propagation characteristics shown by the WG coupler involve all the adiabatic states instead of dark states only.

Keywords: Coherent population transfer and trapping, four-waveguide directional coupler, tripod system, adiabatic states.

1. Introduction

During past few decades, some researchers have focused their attention to investigate the probability of exploration of the microscopic quantum phenomena in optical regime. It provides ample advantage of directly mapping the evolution of wave function in space by simple fluorescence imaging or scanning tunneling optical microscopy. The field opto-quantum analogy has become interesting day by day opening up a new avenue with the emergence of coherent laser source and integrated fiber optics. The opto-quantum analogy have been extended to the field of quantum optics to emulate many interesting phenomena like strongly driven two-level system and Rabi oscillations, Electromagnetically Induced Transparency, Fano Interference, Stabilization of Atoms in Ultra Strong Laser Fields, Stimulated Raman Adiabatic Passage (STIRAP)linked with Coherent Population Trapping (CPT), quantum state embedded in a continuum accompanied by Fano Resonance, quantum information processing and quantum teleportation in photonic structures. WG based photonic structures are frequently used in this quest since optical analogue of laser-matter interaction effects can be effortlessly produced by simple engineering of the guiding structure [1,2]. The direct mapping and space visualization of ultrafast time dependent phenomena is feasible using coupled WG and fiber structure. WG array and directional coupler have been proven to be important optical tools for reproducing microscopic quantum phenomena. WG array based photonic configurations are also employed to

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investigate quantum phenomena like Coherent Population Transfer [3-6] and Trapping [7,8], Fano Resonance [7], Rabi Oscillation [9] effects in broader scale.

In guided wave communication systems, optical WG directional couplers have promising potential applications including power splitting, switching etc. [10-13]. In the simplest form, directional couplers consist of two WGs which are installed sufficiently close so as to allow a coupling between them. As a result, the interference between the two modal fields leads to the periodical exchange of light energy between the two WGs during propagation [14]. Three or more evanescently coupled WG systems have attracted great research interest towards investigation of sophisticated and intriguing behavior in the field dynamics. The schemes for using three-WG couplers have been introduced by Iwasaki *et al.* [15] and experimentally demonstrated by Peall and Syms [16]. The extensive use of the directional coupler as a switching/modulating device has been theoretically and experimentally shown in various configurations and materials. The switching characteristics are analyzed in the three-WG system by launching an incident beam into either the central WG [17] or an outer WG [18,19] or in both [20,21]. The switching of light is feasible with the adiabatic evolution and the suitable design of variable coupling coefficients [19] *viz.* introducing the counterintuitive order. The adiabatic method is expedient as it needs no specific shape of coupling coefficient or definite system parameters. However, to ensure the adiabatic evolution of normal modes, the coupling profiles must allow sufficient overlapping over a significant spatial extent.

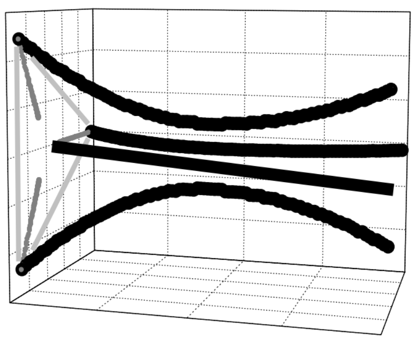

Fig. 1: Schematic presentation of a four-WG directional coupler.

Recently, an adiabatic three-WG coupler is presented in which the initial light is launched in each of the WGs and any sequence of coupling profiles can be allowed to employ [22]. Apart from switching and power splitting, WG directional couplers find applications as self-trapper (complete return of light to the initial state) too [23]. Coherent Population Transfer and CPT in multi-level atomic systems may be envisioned as the quantum analogues of switching, power splitting and self-trapping effects in coupler systems. In most of the previous works on directional couplers, the numbers of WGs were restricted to three only. In our present article, we study the adiabatic light transfer mechanism (optical analogue of coherent population transfer and trapping) in a four-WG directional coupler which assumes tripod like configuration in atomic regime. In almost all earlier works counterintuitive coupling schemes have been used whereas our present study involves any order of coupling schemes and thereby making our approach more general. Also the initial light distribution condition can be varied arbitrarily.

2. Theory and Results

Adiabatic four-WG coupler analogous to a four-level tripod system is presented in Fig.1. In our configuration WG1, WG2 and WG3 are placed to make a small array and the rest WG4 is side coupled with WG2. The coupling coefficients between: WG1 and WG2 is $k_{12}(z)$, WG2 and WG3 is $k_{23}(z)$, WG2 and WG4 is $k_{24}(z)$ whereas coupling between other pairs of WGs have been ignored. We further assume that the propagation constants of all the WGs are equal. In resonant condition, the tripod like four WG system is described by the Hamiltonian

$$H(z)=\hbar\begin{pmatrix} 0 & k_{12} & 0 & 0 \\ k_{12} & 0 & k_{23} & k_{24} \\ 0 & k_{23} & 0 & 0 \\ 0 & k_{24} & 0 & 0 \end{pmatrix} \tag{1}$$

where coupling terms are real and satisfy the relation, $k_{ij}=k_{ji}$. Hamiltonian (represented by Eq. (1)) has four eigen values (two of which are equal) given by $\lambda_1=\lambda_2=0$, $\lambda_3=-\lambda_4=\hbar k(z)$ and $k(z)=\left(k_{12}^2+k_{23}^2+k_{24}^2\right)^{1/2}$. The corresponding adiabatic states can be expressed in terms of two z dependent mixing angles defined as $\tan\vartheta(z)=\dfrac{k_{12}(z)}{\left(k_{23}^2(z)+k_{24}^2(z)\right)^{1/2}}$ and $\tan\varphi(z)=\dfrac{k_{24}(z)}{k_{23}(z)}$.

The adiabatic states corresponding to two zero eigen values (dark states) are

$$\phi_1(z)=\psi_1\cos\vartheta(z)-\psi_3\sin\vartheta(z)\cos\varphi(z)-\psi_4\sin\vartheta(z)\sin\varphi(z) \tag{2}$$

$$\phi_2(z)=\psi_3\sin\varphi(z)-\psi_4\cos\varphi(z). \tag{3}$$

Two remaining adiabatic states are

$$\phi_3(z)=\frac{1}{\sqrt{2}}\left[\psi_1\sin\vartheta(z)+\psi_2+\psi_3\cos\vartheta(z)\cos\varphi(z)+\psi_4\cos\vartheta(z)\sin\varphi(z)\right] \tag{4}$$

$$\phi_4(z)=\frac{1}{\sqrt{2}}\left[\psi_1\sin\vartheta(z)-\psi_2+\psi_3\cos\vartheta(z)\cos\varphi(z)+\psi_4\cos\vartheta(z)\sin\varphi(z)\right] \tag{5}$$

in this regard an interesting point to highlight is that our four WG system is capable of generating double dark resonance. The amplitudes in the original and adiabatic bases are linked through the relation $\phi(z)=M(z)\psi(z)$ where the propagation matrix $M(z)$ is orthogonal and assumes the form

$$M(z)=\begin{pmatrix} \cos\vartheta & 0 & -\sin\vartheta\cos\varphi & -\sin\vartheta\sin\varphi \\ 0 & 0 & \sin\varphi & -\cos\varphi \\ \frac{1}{\sqrt{2}}\sin\vartheta & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\cos\vartheta\cos\varphi & \frac{1}{\sqrt{2}}\cos\vartheta\sin\varphi \\ \frac{1}{\sqrt{2}}\sin\vartheta & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\cos\vartheta\cos\varphi & \frac{1}{\sqrt{2}}\cos\vartheta\sin\varphi \end{pmatrix} \tag{6}$$

The evolution matrix $U^A(z_f,z_i)$ in the adiabatic basis relates initial and final states such that $\phi(z_f)=U^A(z_f,z_i)\phi(z_i)$ where z_i and z_f represent the input and output z coordinates respectively. The evolution matrix in original basis reads as

$$U(z_f,z_i)=M^{-1}(z_f)U^A(z_f,z_i)M(z_i) \tag{7}$$

where

$$U^A(z_f,z_i)=\begin{pmatrix} e^{i\eta_1} & 0 & 0 & 0 \\ 0 & e^{i\eta_2} & 0 & 0 \\ 0 & 0 & e^{i\eta_3} & 0 \\ 0 & 0 & 0 & e^{i\eta_4} \end{pmatrix}$$ contains the phase factors in its diagonal elements.

The different elements of the evolution matrix can be explicitly written as:

$$U_{11}=\cos\vartheta_f\cos\vartheta_i e^{i\eta_1}+\frac{1}{2}\sin\vartheta_f\sin\vartheta_i e^{i\eta_3}+\frac{1}{2}\sin\vartheta_f\sin\vartheta_i e^{i\eta_4}$$

$$U_{12}=\frac{1}{2}\sin\vartheta_f e^{i\eta_3}-\frac{1}{2}\sin\vartheta_f e^{i\eta_4}$$

$$U_{13} = -\cos\vartheta_f \sin\vartheta_i \cos\varphi_i e^{i\eta_1} + \frac{1}{2}\sin\vartheta_f \cos\vartheta_i \cos\varphi_i e^{i\eta_3} + \frac{1}{2}\sin\vartheta_f \cos\vartheta_i \cos\varphi_i e^{i\eta_4}$$

$$U_{14} = -\cos\vartheta_f \sin\vartheta_i \sin\varphi_i e^{i\eta_1} + \frac{1}{2}\sin\vartheta_f \cos\vartheta_i \sin\varphi_i e^{i\eta_3} + \frac{1}{2}\sin\vartheta_f \cos\vartheta_i \sin\varphi_i e^{i\eta_4}$$

$$U_{21} = \frac{1}{2}\sin\vartheta_i e^{i\eta_3} - \frac{1}{2}\sin\vartheta_i e^{i\eta_4}$$

$$U_{22} = \frac{1}{2}e^{i\eta_3} + \frac{1}{2}e^{i\eta_4}$$

$$U_{23} = \frac{1}{2}\cos\vartheta_i \cos\varphi_i e^{i\eta_3} - \frac{1}{2}\cos\vartheta_i \cos\varphi_i e^{i\eta_4}$$

$$U_{24} = \frac{1}{2}\cos\vartheta_i \sin\varphi_i e^{i\eta_3} - \frac{1}{2}\cos\vartheta_i \sin\varphi_i e^{i\eta_4}$$

$$U_{31} = -\cos\vartheta_i \sin\vartheta_f \cos\varphi_f e^{i\eta_1} + \frac{1}{2}\sin\vartheta_i \cos\vartheta_f \cos\varphi_f e^{i\eta_3} + \frac{1}{2}\sin\vartheta_i \cos\vartheta_f \cos\varphi_f e^{i\eta_4}$$

$$U_{32} = \frac{1}{2}\cos\vartheta_f \cos\varphi_f e^{i\eta_3} - \frac{1}{2}\cos\vartheta_f \cos\varphi_f e^{i\eta_4}$$

$$U_{33} = \sin\vartheta_f \cos\varphi_f \sin\vartheta_i \cos\varphi_i e^{i\eta_1} + \sin\varphi_f \sin\varphi_i e^{i\eta_2} + \frac{1}{2}\cos\vartheta_f \cos\varphi_f \cos\vartheta_i \cos\varphi_i e^{i\eta_3}$$

$$+\frac{1}{2}\cos\vartheta_f \cos\varphi_f \cos\vartheta_i \cos\varphi_i e^{i\eta_4}$$

$$U_{34} = \sin\vartheta_f \cos\varphi_f \sin\vartheta_i \sin\varphi_i e^{i\eta_1} - \sin\varphi_f \cos\varphi_i e^{i\eta_2} + \frac{1}{2}\cos\vartheta_f \cos\varphi_f \cos\vartheta_i \sin\varphi_i e^{i\eta_3}$$

$$+\frac{1}{2}\cos\vartheta_f \cos\varphi_f \cos\vartheta_i \sin\varphi_i e^{i\eta_4}$$

$$U_{41} = -\cos\vartheta_i \sin\vartheta_f \sin\varphi_f e^{i\eta_1} + \frac{1}{2}\sin\vartheta_i \cos\vartheta_f \sin\varphi_f e^{i\eta_3} + \frac{1}{2}\sin\vartheta_i \cos\vartheta_f \sin\varphi_f e^{i\eta_4}$$

$$U_{42} = \frac{1}{2}\cos\vartheta_f \sin\varphi_f e^{i\eta_3} - \frac{1}{2}\cos\vartheta_f \sin\varphi_f e^{i\eta_4}$$

$$U_{43} = \sin\vartheta_i \cos\varphi_i \sin\vartheta_f \sin\varphi_f e^{i\eta_1} - \sin\varphi_i \cos\varphi_f e^{i\eta_2} + \frac{1}{2}\cos\vartheta_i \cos\varphi_i \cos\vartheta_f \sin\varphi_f e^{i\eta_3}$$

$$+\frac{1}{2}\cos\vartheta_i\cos\varphi_i\cos\vartheta_f\sin\varphi_f\, e^{i\eta_4}$$

$$U_{44} = \sin\vartheta_f\sin\varphi_f\sin\vartheta_i\sin\varphi_i\, e^{i\eta_1} + \cos\varphi_f\cos\varphi_i\, e^{i\eta_2} + \frac{1}{2}\cos\vartheta_f\sin\varphi_f\cos\vartheta_i\sin\varphi_i\, e^{i\eta_3}$$

$$+\frac{1}{2}\cos\vartheta_f\sin\varphi_f\cos\vartheta_i\sin\varphi_i\, e^{i\eta_4}$$

Let us assume $z_i = -\infty$ and $z_f = +\infty$

2.1 We now consider four distinct cases of coupling arrangements:

2.1.1 Case 1

The coupling coefficients are arranged in a manner such that $k_{23}(z)$ originates before and terminates after $k_{12}(z)$ whereas $k_{24}(z)$ is delayed with respect to both $k_{23}(z)$ and $k_{12}(z)$. Following Asymptotic relations are applicable for the aforementioned coupling sequence: $\vartheta_i(z=-\infty) = \vartheta_f(z=+\infty) = \phi_i(z=-\infty) = 0, \phi_f(z=+\infty) = \frac{\pi}{2}$.Considering such arrangement we take four different cases of initial light distributions.

(i) If the initial amplitude is given by the matrix $(1,0,0,0)^T$, the final light amplitude will be $(e^{i\eta_1},0,0,0)^T$ which is clearly a self-trapping case.

(ii) When the initial amplitude is $(0,1,0,0)^T$, final light amplitude will be $(0,\frac{1}{2}(e^{i\eta_3}+e^{i\eta_4}),0,\frac{1}{2}(e^{i\eta_3}-e^{i\eta_4}))^T$. It is evident that initial light in the 2nd WG will be divided into the 2nd and 4th WGs and the amount of light energy present in these WGs depends upon the phase factors η_3 and η_4. In particular for $\eta_3 = \eta_4$, the light will be completely trapped in the2nd WG (self-trapping case).

(iii) Elementary light amplitude $(0,0,1,0)^T$ leads to the final amplitude given by $(0,\frac{1}{2}(e^{i\eta_3}-e^{i\eta_4}),0,\frac{1}{2}(e^{i\eta_3}+e^{i\eta_4}))^T$. Thus initial light in the 3rd WG will be shared into the 2nd and 4th WGs. In particular when, $\eta_3 = \eta_4$, the light will be completely transferred from the 3rd WG to 4th WG. Thus it is obviously a case of light switching.

(iv) If the initial light distribution is $(0,0,0,1)^T$, it leads to a final light distribution given by $(0,0,-e^{i\eta_2},0)^T$ which is a case of power switching from the 4th to the 3rd WG.

Thus our four WG coupler configuration is capable to exhibit power splitting, self-trapping and switching phenomena under different initial light distribution conditions owing to some specific system parameters.

2.1.2 Case 2

We may engineer the coupling coefficients in a manner in which $k_{23}(z)$ precedes $k_{24}(z)$ and $k_{24}(z)$ precedes $k_{12}(z)$. Such sequence of coupling can be mathematically represented by the following asymptotic relations:

$$\vartheta_i(z=-\infty)=0, \vartheta_f(z=+\infty)=\frac{\pi}{2}, \phi_i(z=-\infty)=0, \phi_f(z=+\infty)=\frac{\pi}{2}.$$

(i) Now if we envisage the elementary amplitude as $(1,0,0,0)^T$, the final light amplitude will be $(0,0,0,-e^{i\eta_1})^T$. It is clearly a switching case in which light is absolutely transferred from 1st to 4th WG.

(ii) When the initial light sharing is given as $(0,1,0,0)^T$, ultimate light amplitude will be represented by $(0,\frac{1}{2}(e^{i\eta_3}-e^{i\eta_4}),0,\frac{1}{2}(e^{i\eta_3}+e^{i\eta_4}))^T$. It is evident that initial light in the 2nd WG will be shared into the 2nd and4th WGs and the phase factors η_3 and η_4 regulates the light energy content in these WGs. Specifically, for $\eta_3=\eta_4$, the light will be completely shifted into the 4th WG (switching case).

(iii) Final light amplitude will be $(0,\frac{1}{2}(e^{i\eta_3}+e^{i\eta_4}),0,\frac{1}{2}(e^{i\eta_3}-e^{i\eta_4}))^T$ for initial light distribution $(0,0,1,0)^T$. Thus initial light in the 3rd WG will be split into the 2nd and 4th WGs. For $\eta_3=\eta_4$, the light will be completely transferred from the 3rd WG to the 2nd WG.

(iv) If the initial light sharing is $(0,0,0,1)^T$, it directs to a final energy distribution $(0,0,-e^{i\eta_2},0)^T$ which is the case of power switching from the 4th to the 3rd WG.

2.1.3 Case 3

As an alternative, we may fix up the coupling coefficients in a way in which $k_{24}(z)$ precedes $k_{23}(z)$ and $k_{23}(z)$ precedes $k_{12}(z)$. This physical condition may be represented in terms of the following asymptotic relations:

$\vartheta_i(z=-\infty)=0, \vartheta_f(z=+\infty)=\frac{\pi}{2}, \phi_i(z=-\infty)=\frac{\pi}{2}, \phi_f(z=+\infty)=0$.

(i) Under such order of coupling coefficients, if the initial amplitude is $(1,0,0,0)^T$, the final light amplitude will be $(0,0,-e^{i\eta_1},0)^T$. It is clearly a switching case in which light is entirely transferred from the 1st to the 3rd WG.

(ii) Final light amplitude takes the form $(\frac{1}{2}(e^{i\eta_3}-e^{i\eta_4}),\frac{1}{2}(e^{i\eta_3}+e^{i\eta_4}),0,0)^T$ for the initial amplitude $(0,1,0,0)^T$. It is manifested that initial light in the 2nd WG will be split into the 2nd and 1st WG and the percentage of light in these WGs figures on the phase factors η_3 and η_4. Perfect trapping of light in the 2nd WG occurs particularly for $\eta_3=\eta_4$ (self-trapping case).

(iii) For elemental light amplitude $(0,0,1,0)^T$, final amplitude will be $(0,0,0,-e^{i\eta_2})^T$ which indicates that there will be complete switching of light from the 3rd to the 4th WG.

(iv) When we employ the initial amplitude as $(0,0,0,1)^T$, final light amplitude assumes the form $(\frac{1}{2}(e^{i\eta_3}+e^{i\eta_4}),\frac{1}{2}(e^{i\eta_3}-e^{i\eta_4}),0,0)^T$. It is clear that initial light in the 4th WG will be split into the 2nd and 1st WGs and the content of light energy in these WGs depends upon the phase factors η_3 and η_4. In particular for $\eta_3=\eta_4$, the light will be completely shifted into the 1st WG (switching case).

2.1.4 Case 4

In fine, we engineer the coupling coefficients in a manner that the coupling coefficients $k_{24}(z)$ and $k_{23}(z)$ coincide and precede $k_{12}(z)$. Such type of coupling sequences satisfy the following asymptotic relations: $\vartheta_i(z=-\infty)=0, \vartheta_f(z=+\infty)=\frac{\pi}{2}, \phi_i(z=-\infty)=\phi_f(z=+\infty)=\frac{\pi}{4}$.

(i) If we introduce the initial amplitude as $(1,0,0,0)^T$, the final light amplitude will be $(0,0,-\frac{1}{\sqrt{2}}e^{i\eta_1},-\frac{1}{\sqrt{2}}e^{i\eta_1})^T$. It is clearly a power splitting case in which light in the 1st WG is equally transferred to the 3rd and 4th WGs.

(ii) Elementary light amplitude $(0,1,0,0)^T$ leads the final amplitude $(\frac{1}{2}(e^{i\eta_3}-e^{i\eta_4}),\frac{1}{2}(e^{i\eta_3}+e^{i\eta_4}),0,0)^T$. Obviously initial light in the 2nd WG will be shared into the 2nd and 1st WGs and the amount of light in these WGs is ruled by the phase factors η_3 and η_4. In particular for $\eta_3 = \eta_4$, the light will be completely trapped in the 2nd WG (self-trapping case).

(iii) When the initial amplitude is $(0,0,1,0)^T$, final light amplitude will be $(\frac{1}{2\sqrt{2}}(e^{i\eta_3}+e^{i\eta_4}), \frac{1}{2\sqrt{2}}(e^{i\eta_3}-e^{i\eta_4}), \frac{1}{2}e^{i\eta_2}, -\frac{1}{2}e^{i\eta_2})^T$ which indicates that initial light in the 3rd WG will be shifted into all the WGs with equal amount of light in the 3rd and 4th WGs (25% in each). Percentage of light shared by the 1st and 2nd WGs figures on the phase factors η_3 and η_4. For $\eta_3 = \eta_4$, 50% (0%) light will be trapped in the 1st (2nd) WG.

(iv) Final light amplitude will be $(\frac{1}{2\sqrt{2}}(e^{i\eta_3}+e^{i\eta_4}), \frac{1}{2\sqrt{2}}(e^{i\eta_3}-e^{i\eta_4}), -\frac{1}{2}e^{i\eta_2}, \frac{1}{2}e^{i\eta_2})^T$ if we incorporate the initial light amplitude as $(0,0,0,1)^T$. Clearly elemental light in the 3rd WG will be split into all the WGs and the content of light in the 3rd and 4th WGs be equal (25% in each). Percentage of light shared by the 1st and 2nd WGs is governed by the phase factors η_3 and η_4. Specifically for $\eta_3 = \eta_4$, the 50% (0%) light will be trapped in the 1st (2nd) WG.

3. Conclusion

To summarize, the propagation dynamics of a four-WG directional coupler configuration similar to a tripod system in atomic scale is investigated in order to display the optical analogue of coherent population transfer and trapping. Our approach may be treated as a general one in the sense that we have used all the adiabatic states instead of only the dark states (STIRAP technique) responsible for light transfer mechanism. Another important point to highlight is that our system is potent to exhibit double dark resonance. Depending upon the arrangement of various coupling coefficients and different conditions of initial light distribution, the coupler may behave as power splitter, switch and self-trapper etc.

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